Week 11:
Relative extrema, Higher derivatives, Curve sketching and Absolute extrema

11/3

Group 1

Find all relative extrema for the following functions, as well as where each function is increasing and decreasing.

1. $f(x) = 2x^3 - 3x^2 - 72x + 15$

2. $f(x) = 6x^{2/3} - 4x$

3. $f(x) = x^3 e^x$

Group 2

A study on optimizing revenue from a website considered dividing customers into two groups based on a value x between 0 and 1, where x measures the proportion of the total bandwidth requested by a customer. Customers with a request less than x were considered low revenue, and those above x high revenue. The expected revenue from the low revenue customers was described by

$$R(x) = Cx(1 - e^{-kx})$$

where C, k are positive constants.
(a) Find $R'(x)$ and use it to find value of $x \in [0, 1]$ the revenue is increasing.
(b) Find R" and find for which $x \in [0, 1]$ the function is concave up.

Group 3

Sketch the curve

$$f(x) = -2x^3 - 9x^2 + 108x - 10$$

Group 4

A marshy region used for agricultural drainage has become contaminated with selenium. It has been determined that flushing the area with clean water will reduce the selenium for a while, but it will then begin to build up again. A biologist has found that the percent of selenium in the soil x months after the flushing begins is given by

$$f(x) = \frac{x^2 + 36}{2x}, 1 \le x \le 12$$

When will the selenium be reduced to a minimum? What is the minimum percent?

Group 5

ind the absolute extrema if they exist, as well as all values of x where they occur, for each function, and specified domain.
(a) $f(x) = x^3 - 2x^2 - 15x + 10; x \in [-2, 10]$
(b) $f(x) = x + e^{-2x}; [-2, 3]$

Extra Problems

1. Find the absolute extrema if they exist, as well as all values of x where they occur.
   (a) $f(x) = 4x + \frac{54}{x^2} + 2, \ x > 0$
   (b) $f(x) = x \ln x, \ x > 0$

2. Sketch the curve
$$f(x) = (e^x + e^{-x})/2$$

3. Find the second derivative of the following functions:
   (a) $f(x) = x^3 + 6x^2 - 9x - 8$
   (b) $f(x) = (x^2 - 1)^2$
   (c) $f(x) = 4x \ln x$
   (d) $f(x) = x/e^x$